



FACULTY OF SCIENCE AND ENGINEERING

ACADEMIC YEAR 2020/21

| | |
|---------------|--|
| Module Code: | SE4003 |
| Module Title: | Mathematics for Scientists and Engineers |
| Level: | 4 |
| Date: | 27 th April 2021 |
| Time: | 9:30 am |
| Duration: | 24 HOURS |

INSTRUCTIONS TO CANDIDATES:

Answer ALL TWENTY questions. Each question is worth 5 marks.

You can look-up anything in your textbook or other valid online resources. You can submit a pdf document created in MS Word containing typed text/equations or scanned hand-written solutions or a combination of both.

Some of the questions contain parameters A, B, C etc. that are specific to each person taking the exam. You need to look-up the parameters which are given for your Assessment Number in the table at the end of the paper. If you cannot find your Assessment Number please email Steve Wilkinson s.j.wilkinson@chester.ac.uk as soon as you can.

Some of the questions involve scientific or engineering words that you may not be familiar with but do not worry if you do not understand them, just focus on the mathematics which you should be able to do.

Part A

In the design of engineering control systems, a key concept is that of the transfer function which is related to the dynamics of the system – i.e. how it changes over time.

An aircraft control system is characterised by the following transfer function:

$$y(s) = \frac{g(s)}{f(s)} \text{ where } f(s) = s^3 + As^2 + Bs + C \text{ and } g(s) = Ds + E$$

Where the parameters A, B, C, D, E are given for each student in the table at the end of this exam.

1. Factorise $f(s)$ above given that one of the factors is $s - 3$ and hence give all the poles of $y(s)$.

[5 marks]

2. Find the derivative of $f(s)$ and hence find the values of s at the local minimum value and the local maximum value of $f(s)$ respectively.

[5 marks]

3. Using your answer from Question 1, express the function $y(s)$ as the sum of partial fractions. If you are unable to find an answer to Question 1 use the alternative expression given below:

$$y(s) = \frac{g(s)}{f(s)} = \frac{3s - 33}{(s + 1)(s - 3)(s - 5)}$$

[5 marks]

4. Using your answer from Question 3, evaluate the indefinite integral of the function $y(s)$ with respect to s . If you are unable to find an answer to Question 3. use the alternative expression given below.

$$y(s) = \frac{2}{(s + 6)} - \frac{1}{3(s + 4)} - \frac{2}{5(s - 2)}$$

[5 marks]

Part B

A search drone has a safe range of A km before needing to return to base to be recharged.

5. A fully charged drone, travelling at a constant altitude, sets off from base and follows a bearing of 50° for B km then switches to a new bearing of 180° . How far can it travel on the new bearing before returning to base and what bearing should it follow? [Note: this is a difficult question and you will get some marks for drawing a correct diagram and writing the correct

equations for the unknowns. You can use trial and error to get an answer as long as you show your working]

[5 marks]

6. Calculate the magnitude and direction of net thrust/lift force required to keep the drone in Question 5 moving at a steady cruising velocity if the weight of the drone is A N and the horizontal drag force it experiences is B N.

[5 marks]

Part C

Solutions of long polymer chains are often pseudoplastic (or shear-thinning fluids) which give a large resistance to flow at low shear because they are like loose 'balls of string' which can be entangled. At high shear they become stretched along the flow and offer less resistance. The effective viscosity μ_{eff} of a pseudoplastic fluid is given by the following equation:

$$\mu_{\text{eff}} = K\dot{\gamma}^{n-1}$$

Where:

- μ_{eff} is the effective viscosity (SI units: Pa s)
- n is known as the flow behaviour index (dimensionless)
- $\dot{\gamma}$ is the shear rate (SI units: s^{-1})
- K is known as the flow consistency index

7. Showing your working, use the equation above to find the SI units of the flow consistency index K .

[5 marks]

8. An experiment is performed on a pseudoplastic fluid in which the effective viscosity is measured for varying shear rates as given in the table below. By plotting an appropriate graph and fitting a straight line, find the values of n and K for this fluid. Give your answers to 3 significant figures. Note that, if you are able to, you should plot the data in Excel using a scatter chart with a linear trend-line.

| | | | |
|---|----|-----|-----|
| shear rate $\dot{\gamma}$ (s^{-1}) | 50 | 200 | 500 |
| effective viscosity μ_{eff} (Pa s) | A | B | C |

[5 marks]

Part D

A fundamental particle is constrained to move in one dimension and its displacement with time t is given by the function $y(t)$.

9. Write down the mathematical expressions for the following:

- a. The dependent variable
- b. The independent variable
- c. The velocity of the particle
- d. The acceleration of the particle

[5 marks]

10. Determine $\frac{d^2y}{dt^2}$ where $\frac{dy}{dt} = e^{2t}\sin(t)$ using the product rule.

[5 marks]

11. Evaluate $\frac{dy}{dt}$ at $t = 1$ where $y(t) = \sqrt{t + \cos(t)}$ using the chain rule. Give your answer to 3 significant figures.

[5 marks]

12. Determine $\frac{dy}{dt}$ where $y(t) = \frac{\ln(t+1)}{t^2+t}$ using the quotient rule.

[5 marks]

13. The velocity v of the fundamental particle introduced earlier varies as a function of time. If $v = 0.6t\sqrt{0.2t^2 + 1}$ write down an expression for the distance travelled between $t = 3$ seconds and $t = 4$ seconds. (Hint: this expression will be a definite integral). Evaluate the value of the integral using an appropriate substitution giving your answer to 3 significant figures.

[5 marks]

14. Evaluate the value of the following integral I using integration by parts:

$$I = \int_0^2 (2x + 1)e^{-\frac{x}{2}} dx$$

[5 marks]

15. Approximate the value of the integral I given in Question 14 above by using the Trapezium Rule with two strips ($h = 1$). Give your answer to 3 significant figures. Compare this value with the exact value from your answer to Question 14 and comment on the reason for the difference. (HINT: you could plot the integrand in Excel to see its shape in the region of interest). If you cannot find an answer for this question or the previous one, you can still comment on whether you expect the Trapezium Rule to give a higher or a lower value than the exact result and explain why you think this.

[5 marks]

Part E

16. Plot the complex numbers $z_1 = A + Bj$ and $z_2 = C + Dj$ on an Argand diagram and evaluate the following while they are in Cartesian form:

- a. $z_2 - 2z_1$

b. z_1/z_2

[5 marks]

17. Show the following relationship using De Moivre's theorem stating any other trigonometric identities that you use:

$$\cos(2t) = 2\cos^2(t) - 1$$

[5 marks]

18. Find the values of the complex number that satisfy the equation $z^3 = 1 - j$. Express your answers in polar form.

[5 marks]

Part F

19. For the matrices, $P = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} A & B \\ B & C \end{pmatrix}$, determine the following:

- $P + 3Q - I$ (where I is the 2x2 identity matrix)
- $Q^T P$

[5 marks]

20. A farmer has 13 hectares of land and 5 workers which he uses to farm cows and sheep. Each cow requires 0.1 hectares of land whereas each sheep requires 0.05 hectares. On the other hand, each worker can look after 10 cows or 100 sheep. If the land and the workers are fully occupied, write down equations to calculate the number of cows and sheep respectively. Write the equations in matrix notation and solve them by calculating the inverse of the appropriate 2x2 matrix to find the number of cows and the number of sheep that the farmer must have.

[5 marks]

**END OF QUESTIONS BUT PLEASE SEE PARAMETER TABLE
ON THE NEXT PAGE**

| Assess. Num. | Q1,Q2,Q3 | | | | | Q5 | | Q6 | | Q8 | | | Q16 | | | | Q19 | | |
|-----------------|----------|-----|----|----|------|----|----|----|----|-------|-------|-------|-----|---|---|----|-----|---|----|
| | A | B | C | D | E | A | B | A | B | A | B | C | A | B | C | D | A | B | C |
| J76749 | -7 | 0 | 36 | 7 | -66 | 60 | 13 | 50 | 13 | 0.037 | 0.024 | 0.019 | -1 | 2 | 3 | -2 | 6.5 | 2 | -3 |
| J84411 | -4 | -9 | 36 | 13 | -45 | 85 | 16 | 42 | 18 | 0.036 | 0.022 | 0.016 | -1 | 3 | 4 | -1 | 7 | 3 | -2 |
| J83572 | -7 | 0 | 36 | 18 | -84 | 75 | 14 | 32 | 19 | 0.027 | 0.013 | 0.008 | -1 | 5 | 4 | -3 | 5 | 2 | -3 |
| J76124 | -7 | -9 | 63 | 16 | -72 | 65 | 12 | 40 | 16 | 0.055 | 0.042 | 0.035 | -2 | 3 | 2 | -1 | 7.5 | 3 | -1 |
| J86715 | -4 | -17 | 60 | 25 | -89 | 85 | 14 | 34 | 16 | 0.023 | 0.011 | 0.007 | -1 | 5 | 3 | -1 | 5 | 2 | -1 |
| J83947 | -7 | 7 | 15 | 8 | -40 | 70 | 12 | 36 | 14 | 0.043 | 0.029 | 0.022 | -2 | 3 | 2 | -2 | 7.5 | 3 | -2 |
| J75737 | -6 | 5 | 12 | 12 | -48 | 70 | 18 | 30 | 13 | 0.095 | 0.077 | 0.067 | -3 | 2 | 4 | -2 | 5.5 | 2 | -3 |
| J68582 | -7 | -9 | 63 | -2 | -66 | 70 | 16 | 50 | 11 | 0.061 | 0.050 | 0.043 | -1 | 4 | 4 | -3 | 4 | 2 | -2 |
| J88286 | -8 | 9 | 18 | 1 | -27 | 65 | 9 | 36 | 16 | 0.056 | 0.040 | 0.032 | -1 | 5 | 2 | -3 | 4 | 3 | -1 |
| J76877 | -5 | -18 | 72 | 27 | -102 | 85 | 8 | 40 | 20 | 0.091 | 0.069 | 0.058 | -1 | 4 | 5 | -3 | 4.5 | 1 | -3 |
| J88285 | -6 | -1 | 30 | 15 | -75 | 60 | 15 | 48 | 17 | 0.017 | 0.008 | 0.005 | -2 | 5 | 3 | -2 | 6 | 1 | -2 |
| J75978 | -7 | 0 | 36 | 15 | -90 | 70 | 13 | 36 | 13 | 0.023 | 0.011 | 0.007 | -1 | 4 | 4 | -1 | 7 | 1 | -2 |
| J75975 | -4 | -9 | 36 | 5 | -27 | 60 | 11 | 50 | 16 | 0.034 | 0.022 | 0.017 | -2 | 3 | 4 | -1 | 5 | 1 | -3 |
| J84918 | -4 | -9 | 36 | 11 | -51 | 85 | 16 | 38 | 19 | 0.071 | 0.051 | 0.040 | -3 | 2 | 4 | -1 | 6 | 3 | -2 |
| J84757 | -6 | -9 | 54 | 9 | -81 | 80 | 9 | 44 | 11 | 0.073 | 0.055 | 0.046 | -3 | 3 | 5 | -2 | 7 | 3 | -2 |
| J85119 | -5 | -2 | 24 | 17 | -56 | 75 | 17 | 40 | 19 | 0.059 | 0.039 | 0.029 | -2 | 2 | 3 | -1 | 8 | 3 | -1 |
| J86686 | -8 | 1 | 42 | 5 | -35 | 85 | 16 | 32 | 17 | 0.028 | 0.014 | 0.009 | -1 | 4 | 5 | -3 | 5 | 3 | -2 |
| J85678 | -6 | -19 | 84 | 29 | -115 | 65 | 16 | 40 | 20 | 0.062 | 0.041 | 0.031 | -1 | 3 | 2 | -1 | 5.5 | 3 | -3 |
| J69834 | -3 | -16 | 48 | 15 | -52 | 75 | 18 | 32 | 19 | 0.059 | 0.039 | 0.029 | -3 | 2 | 3 | -1 | 7.5 | 1 | -1 |
| J75970 | -7 | 0 | 36 | 18 | -84 | 85 | 9 | 42 | 19 | 0.055 | 0.042 | 0.035 | -1 | 4 | 3 | -2 | 4.5 | 2 | -3 |
| J79800 | -6 | -9 | 54 | 9 | -81 | 70 | 16 | 32 | 16 | 0.024 | 0.013 | 0.009 | -2 | 5 | 5 | -3 | 6.5 | 2 | -3 |
| J85894 | -7 | -9 | 63 | 22 | -114 | 60 | 9 | 34 | 14 | 0.087 | 0.066 | 0.055 | -3 | 4 | 5 | -3 | 8 | 3 | -3 |
| J83233 | -5 | -9 | 45 | 12 | -60 | 85 | 10 | 42 | 11 | 0.067 | 0.054 | 0.047 | -2 | 5 | 3 | -2 | 5 | 3 | -1 |
| J85856 | -3 | -16 | 48 | 15 | -52 | 80 | 16 | 44 | 15 | 0.021 | 0.011 | 0.007 | -2 | 3 | 2 | -1 | 7 | 2 | -2 |
| J88282 | -7 | 0 | 36 | 10 | -60 | 75 | 17 | 32 | 18 | 0.053 | 0.035 | 0.026 | -2 | 4 | 3 | -3 | 5 | 1 | -3 |
| J84144 | -7 | 0 | 36 | 15 | -90 | 60 | 14 | 30 | 12 | 0.091 | 0.069 | 0.058 | -3 | 5 | 4 | -1 | 7.5 | 3 | -3 |
| J87296 | -9 | 11 | 21 | 12 | -52 | 55 | 16 | 40 | 20 | 0.051 | 0.031 | 0.023 | -3 | 4 | 3 | -3 | 7.5 | 2 | -2 |
| J76570 | -6 | -1 | 30 | 3 | -29 | 65 | 18 | 44 | 10 | 0.073 | 0.055 | 0.046 | -2 | 2 | 3 | -2 | 5.5 | 1 | -3 |
| J86822 | -7 | -9 | 63 | 26 | -102 | 60 | 14 | 46 | 16 | 0.023 | 0.013 | 0.009 | -3 | 5 | 3 | -2 | 5 | 1 | -2 |
| J77784 | -7 | 0 | 36 | 21 | -78 | 70 | 16 | 36 | 12 | 0.023 | 0.011 | 0.007 | -3 | 3 | 2 | -3 | 8 | 3 | -2 |
| J75798 | -4 | -17 | 60 | 5 | -43 | 65 | 17 | 40 | 10 | 0.053 | 0.035 | 0.026 | -3 | 5 | 3 | -3 | 4.5 | 3 | -3 |
| J85850 | -5 | -18 | 72 | 4 | -54 | 80 | 13 | 50 | 13 | 0.045 | 0.032 | 0.025 | -2 | 4 | 5 | -2 | 6.5 | 2 | -1 |
| J83182 | -8 | 9 | 18 | 15 | -69 | 80 | 18 | 36 | 20 | 0.049 | 0.035 | 0.027 | -3 | 2 | 4 | -2 | 4.5 | 3 | -1 |
| J77931 | -7 | -9 | 63 | 12 | -84 | 75 | 11 | 44 | 17 | 0.016 | 0.008 | 0.005 | -1 | 4 | 3 | -3 | 4 | 3 | -1 |
| J79838 | -7 | 0 | 36 | 18 | -84 | 60 | 17 | 40 | 19 | 0.064 | 0.045 | 0.036 | -3 | 2 | 5 | -1 | 5.5 | 3 | -3 |
| J84334 | -8 | 9 | 18 | 15 | -69 | 55 | 9 | 46 | 17 | 0.024 | 0.012 | 0.008 | -1 | 5 | 4 | -2 | 7 | 3 | -3 |
| J87089 | -5 | -2 | 24 | 10 | -40 | 70 | 12 | 32 | 19 | 0.048 | 0.030 | 0.022 | -2 | 2 | 5 | -2 | 6 | 2 | -1 |
| J83446 | -5 | -9 | 45 | 22 | -78 | 75 | 15 | 36 | 13 | 0.029 | 0.016 | 0.010 | -3 | 4 | 4 | -3 | 7 | 1 | -3 |
| J86440 | -8 | 9 | 18 | 8 | -48 | 85 | 16 | 32 | 19 | 0.036 | 0.020 | 0.014 | -3 | 3 | 5 | -3 | 5.5 | 1 | -2 |
| J75546 | -5 | -9 | 45 | 6 | -30 | 55 | 11 | 36 | 18 | 0.018 | 0.009 | 0.006 | -3 | 3 | 5 | -1 | 6.5 | 3 | -2 |
| J87470 | -5 | -9 | 45 | 18 | -90 | 65 | 9 | 38 | 13 | 0.082 | 0.062 | 0.052 | -2 | 4 | 2 | -2 | 4 | 3 | -3 |
| J77858 | -5 | -18 | 72 | 1 | -66 | 55 | 15 | 48 | 16 | 0.036 | 0.020 | 0.014 | -1 | 2 | 5 | -3 | 4.5 | 1 | -1 |
| J77860 | -9 | 11 | 21 | 12 | -84 | 70 | 12 | 46 | 17 | 0.095 | 0.077 | 0.067 | -2 | 3 | 5 | -3 | 7 | 3 | -3 |
| J87679 | -8 | 1 | 42 | 23 | -89 | 80 | 8 | 50 | 20 | 0.027 | 0.016 | 0.011 | -1 | 3 | 4 | -1 | 7.5 | 3 | -3 |
| J85428 | -6 | -9 | 54 | 21 | -99 | 70 | 12 | 32 | 11 | 0.025 | 0.013 | 0.008 | -2 | 4 | 3 | -2 | 6 | 1 | -3 |
| J83627 | -5 | -2 | 24 | 10 | -40 | 75 | 10 | 34 | 16 | 0.031 | 0.018 | 0.012 | -1 | 4 | 5 | -1 | 8 | 3 | -2 |
| J71497 | -3 | -16 | 48 | 15 | -52 | 85 | 9 | 42 | 13 | 0.038 | 0.023 | 0.017 | -3 | 3 | 2 | -3 | 5.5 | 1 | -1 |
| J85891 | -5 | -2 | 24 | 17 | -56 | 75 | 15 | 44 | 12 | 0.029 | 0.017 | 0.012 | -1 | 3 | 5 | -2 | 6 | 3 | -1 |
| J86222 | -9 | 11 | 21 | 4 | -28 | 55 | 13 | 38 | 18 | 0.049 | 0.035 | 0.027 | -1 | 4 | 2 | -1 | 5.5 | 3 | -2 |
| J83962 | -7 | 7 | 15 | 2 | -22 | 85 | 11 | 36 | 15 | 0.034 | 0.018 | 0.012 | -2 | 4 | 5 | -1 | 5 | 3 | -3 |

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